Lab 12: Poles and Zeros Matlab Answer Sheet

Introduction:
The purpose of this lab is to study poles and zeros and their relationship with the function and stability of the system. Additionally, studying how characteristics of the impulse response indicate the stability of a system.

Procedures:
This section include answers to the following questions:

5.1.) For the differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y(t) = \frac{dx}{dt} + 3x(t)$$

a. Find the corresponding transfer function

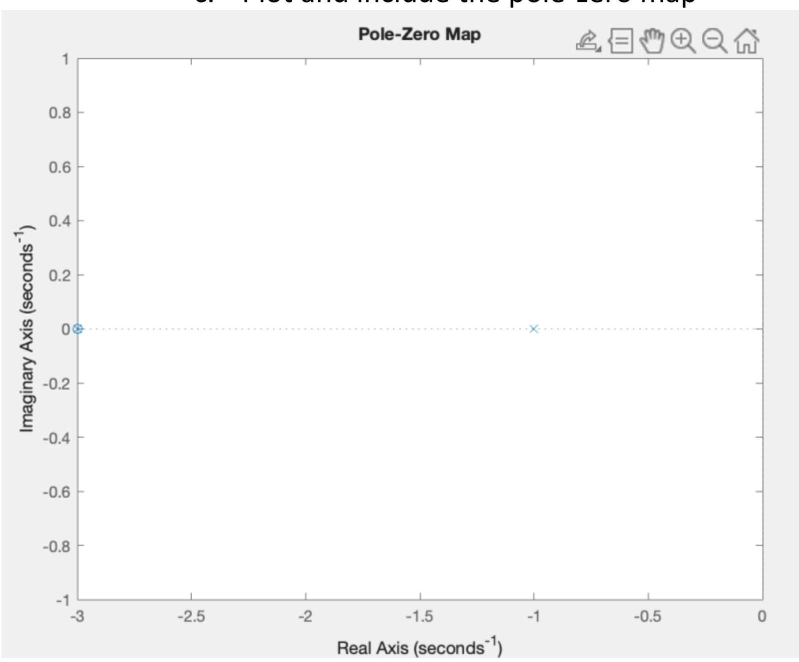
H =

```
    s + 3
-------------
s^2 + 4 s + 3
```

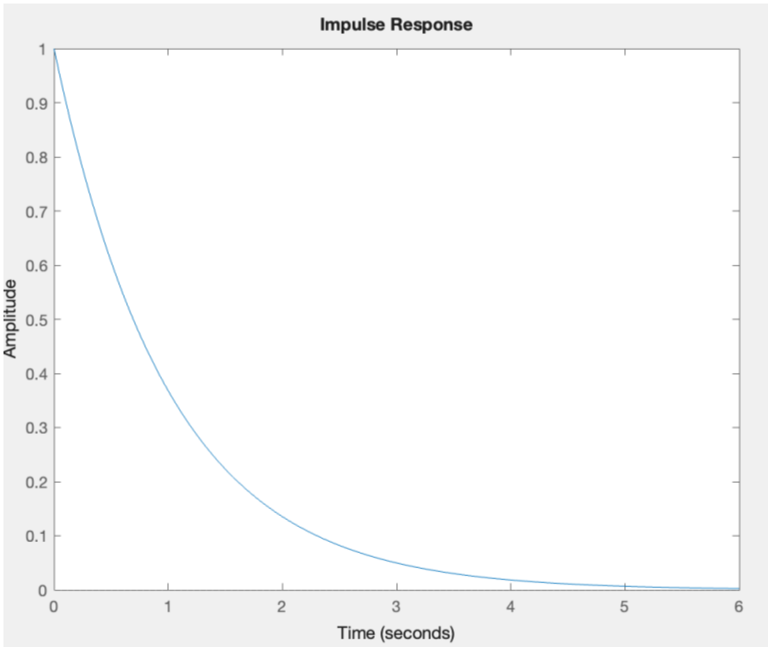
b. Find the poles and zeros

- z = -3; p = -3, -1

c. Plot and include the pole-zero map



d. Define if the system is asymptotically stable, stable, or unstable. Explain using **both** poles and zeros of the system and with the impulse response of the system.



This system is asymptotically stable because its impulse response decays to 0 as time increases. Also, the real part of all poles are strictly less than zero.

5.2.) For the differential equation:

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + \frac{dy}{dt} - 5y(t) = \frac{d^2x}{dt^2} - 7\frac{dx}{dt} + 12x(t)$$

a. Find the corresponding transfer function

H =

s^2 - 7 s + 12

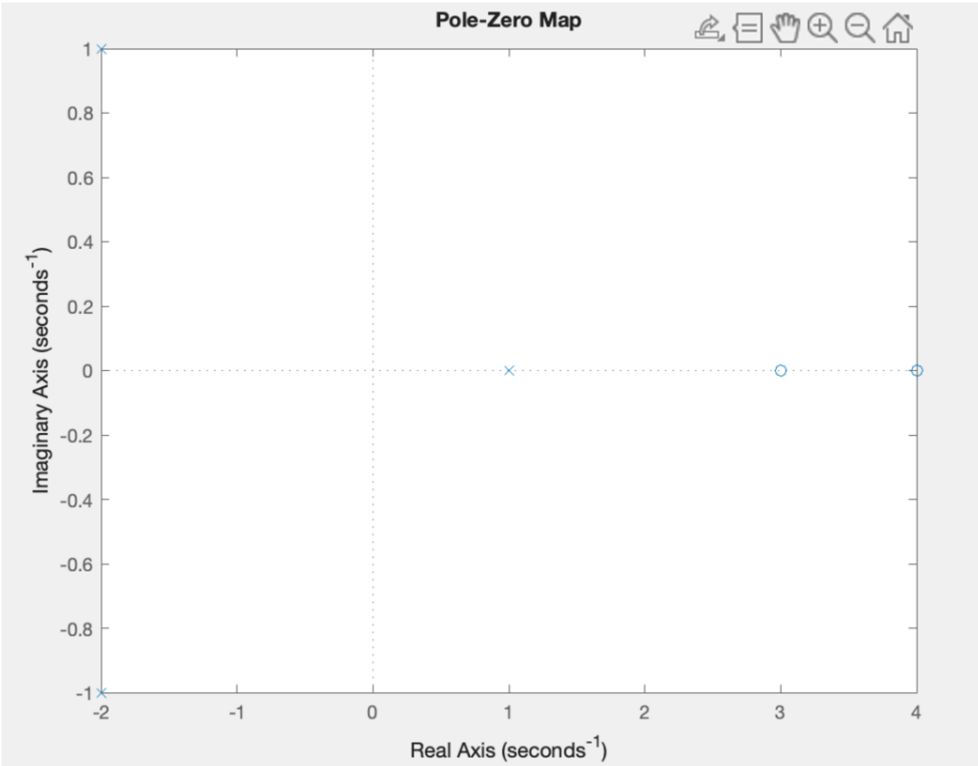
----

s^3 + 3 s^2 + s - 5

b. Find the poles and zeros

- z = 3,4; p = -2 + j, -2 – j, 1

c. Plot and include the pole-zero map



d. Define if the system is asymptotically stable, stable, or unstable. Explain using **both** poles and zeros of the system and with the impulse response of the system.

- Unstable because there is a pole greater than zero
- Unstable because the impulse function increases exponentially as time increases

5.3.) For the differential equation:

$$\frac{d^4y}{dt^4} + 5\frac{d^3y}{dt^3} + 9\frac{d^2y}{dt^2} + 5\frac{dy}{dt} = \frac{d^2x}{dt^2} - 10\frac{dx}{dt} + 21x(t)$$

a. Find the corresponding transfer function

H =

s^2 - 10 s + 21

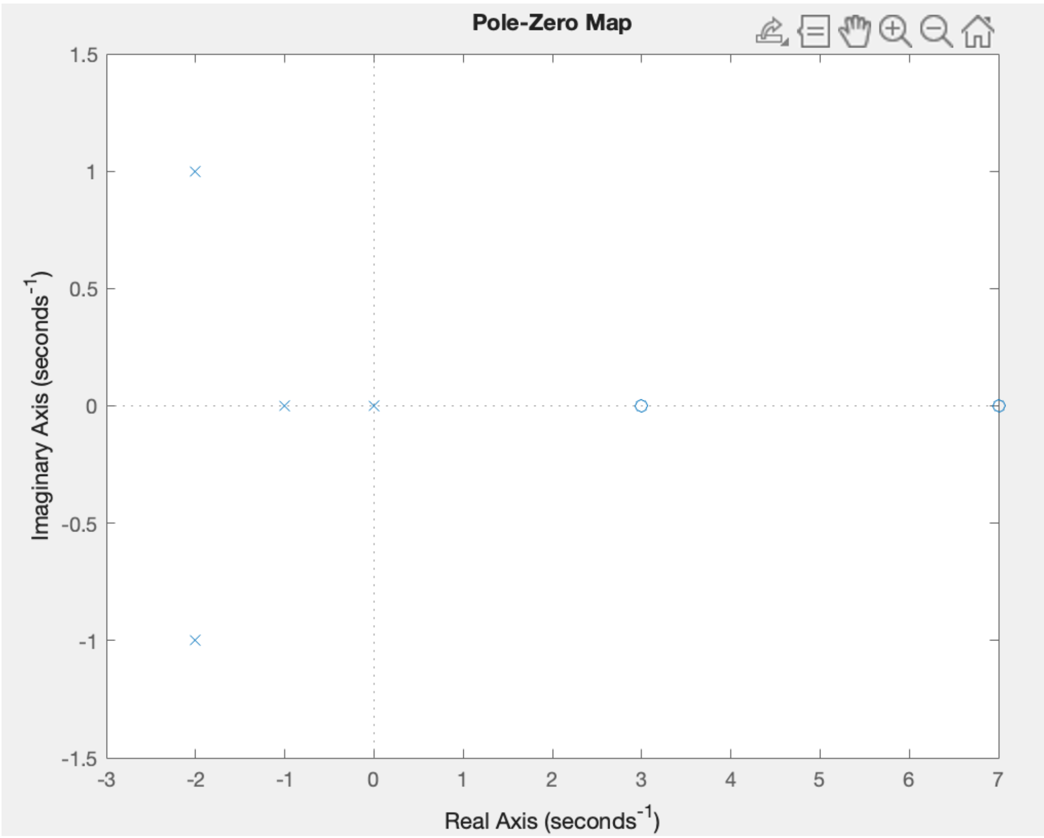
----

s^4 + 5 s^3 + 9 s^2 + 5 s

b. Find the poles and zeros

- z = 3,7; p = 0, -2 + j, -2 – j, -1

c. Plot and include the pole-zero map



d. Define if the system is asymptotically stable, stable, or unstable. Explain using **both** poles and zeros of the system and with the impulse response of the system.

- Stable because all poles equal or less than zero. Not asymptotically because one real pole is exactly zero
- Stable because impulse function approaches a constant, nonzero finite value

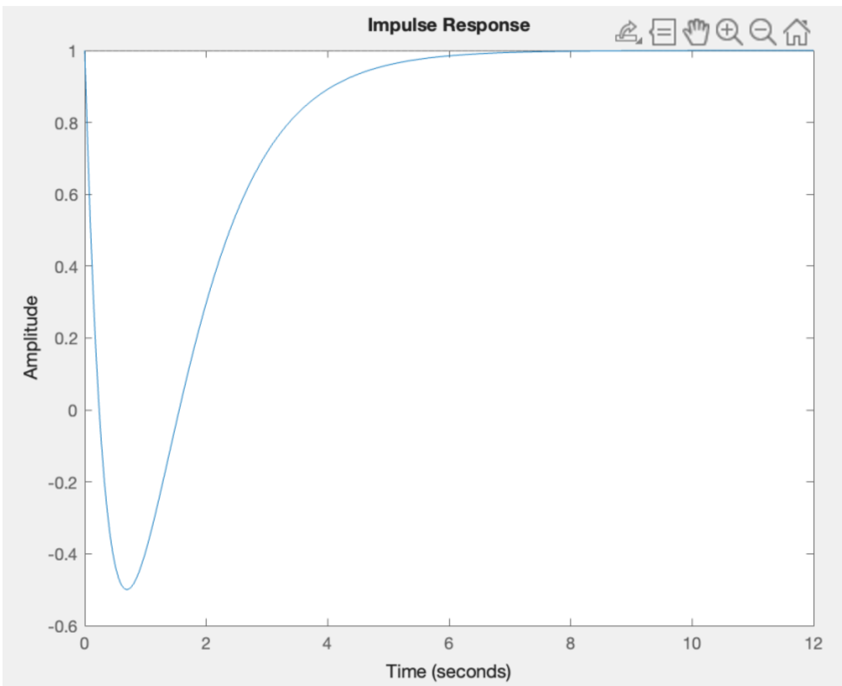
5.4.) Design a third order transfer function that produces a **stable** impulse response. The function should have two zeros.

a. Define your transfer function

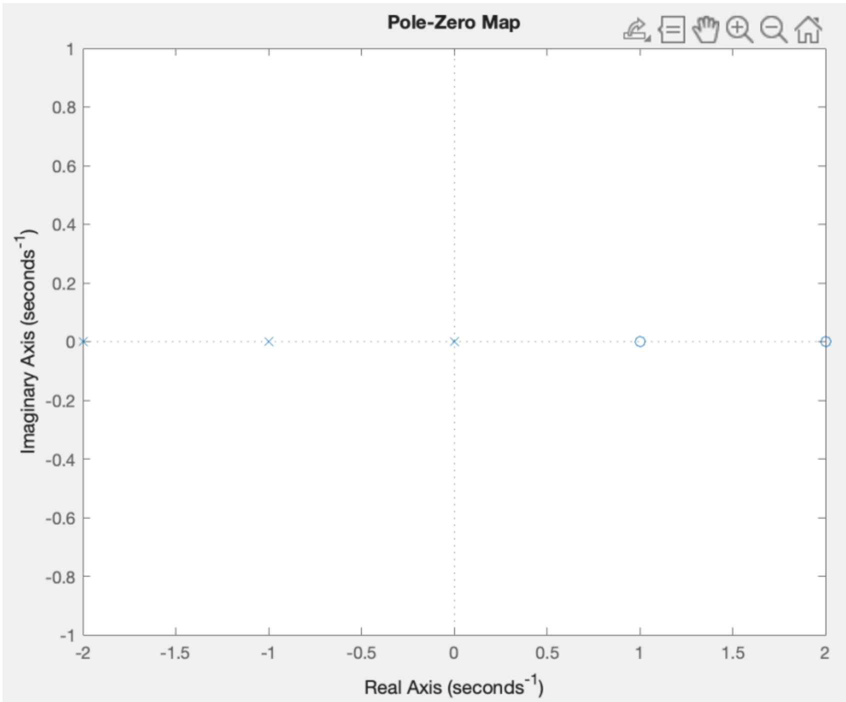
H =

```
  s^2 - 3 s + 2
-----------------
s^3 + 3 s^2 + 2 s
```

b. Plot and include Impulse Response



c. Plot and include Pole-Zero Map



d. Explain why the response is stable.

- Responses is stable because impulse function approaches a constant, nonzero finite value and all poles equal or less than zero. Not asymptotically because one real pole is exactly zero.

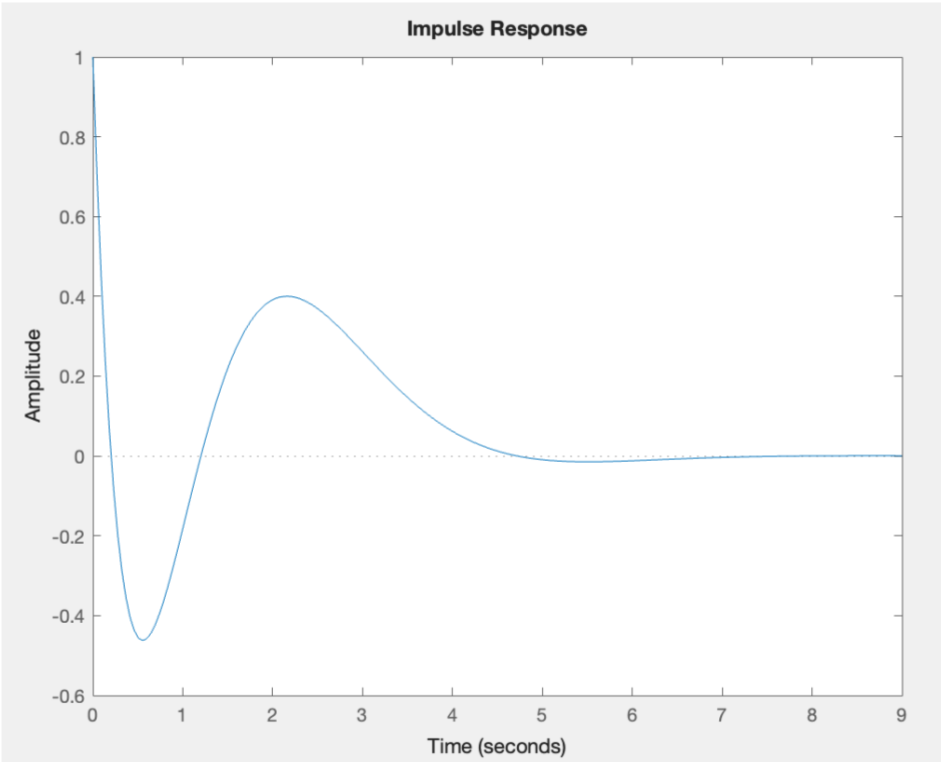
5.5.) Design a third order transfer function that produces an **asymptotically stable** impulse response. The function should have two zeros.

a. Define your transfer function

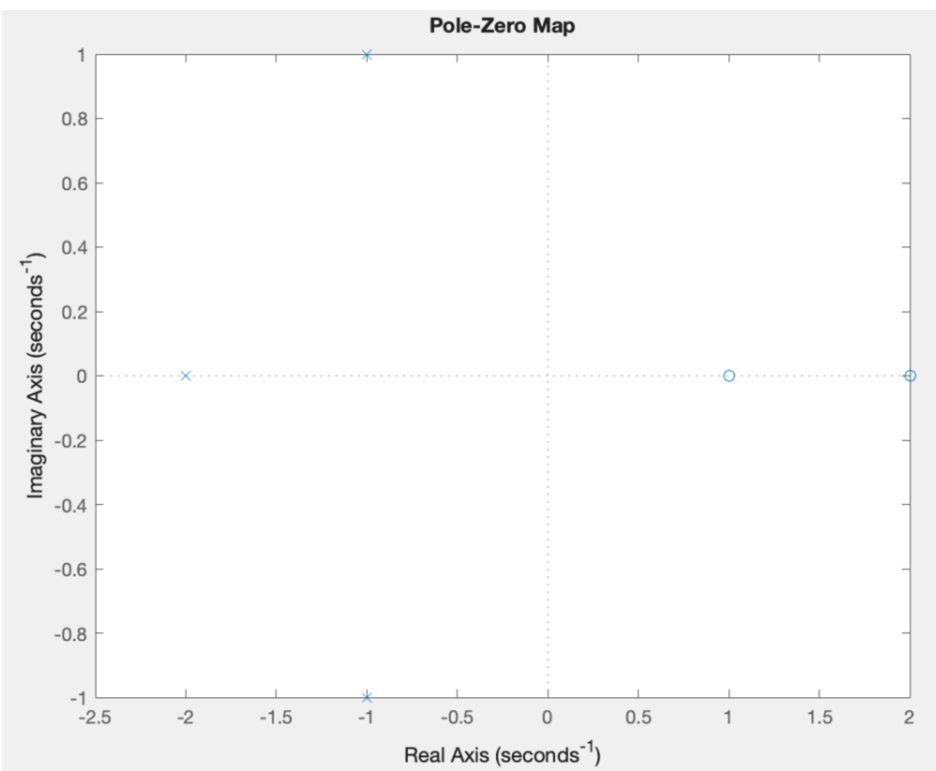
H =

```
   s^2 - 3 s + 2
 ---------------------
 s^3 + 4 s^2 + 6 s + 4
```

b. Plot and include Impulse Response



c. Plot and include Pole-Zero Map



d. Explain why the response is asymptotically stable.

This system is asymptotically stable because its impulse response decays to 0 as time increases. Also, the real parts of all poles are strictly less than zero.

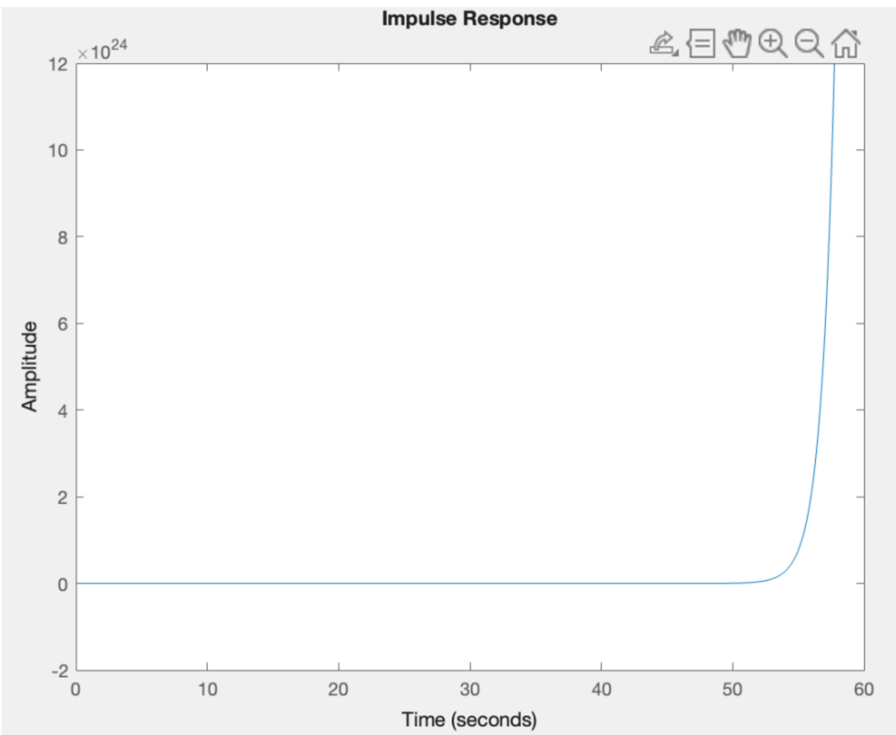
5.6.) Design a third order transfer function that produces an **unstable** impulse response. The function should have two zeros.

a. Define your transfer function

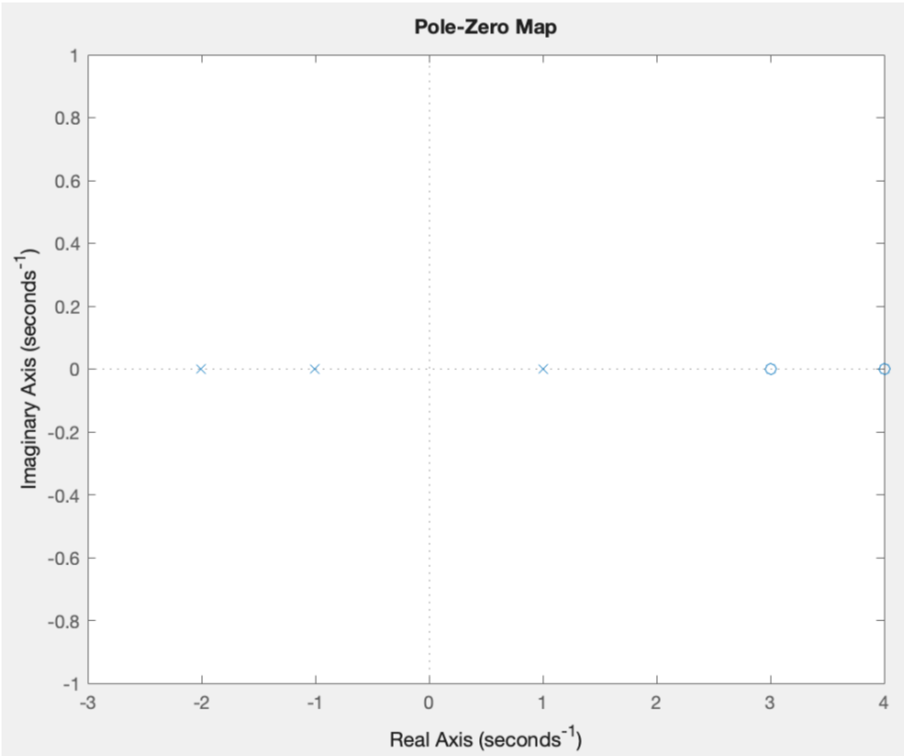
H =

```
  s^2 - 7 s + 12
 -------------------
 s^3 + 2 s^2 - s - 2
```

b. Plot and include Impulse Response



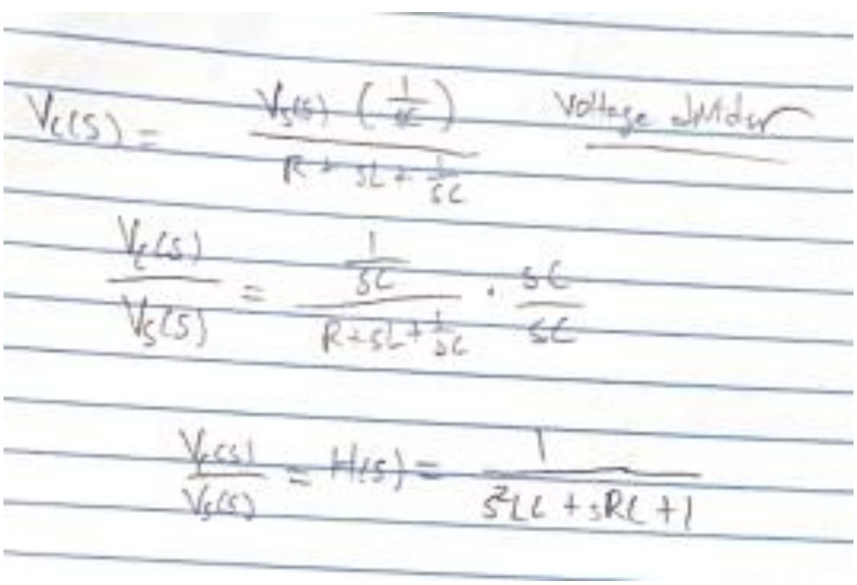
c. Plot and include Pole-Zero Map



d. Explain why the response is unstable.

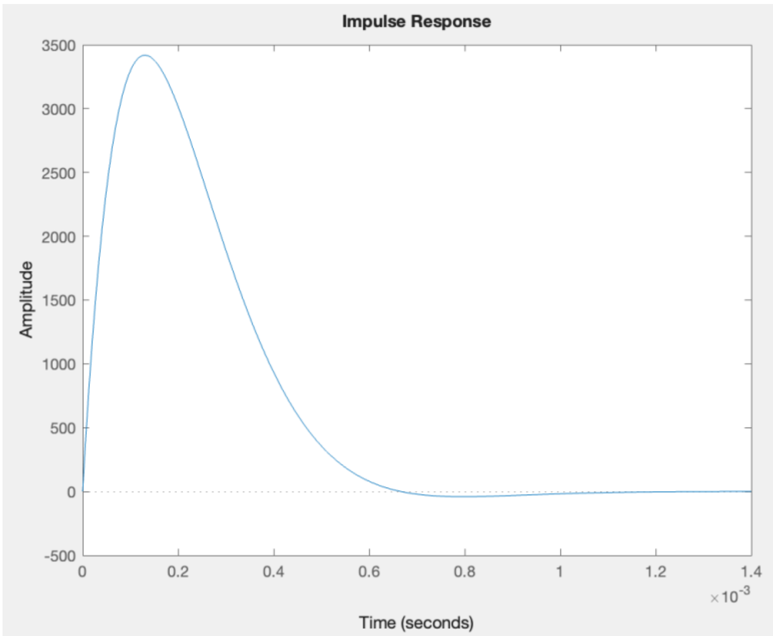
- Unstable because there is a pole greater than zero and because the impulse function increases exponentially as time increases

5.7.) Find the transfer function of the circuit in variables. (Don't plug in any values for the R,L, and C components yet.) Include the transfer function below. Because this formula can be used for any component values, we call this the general form.

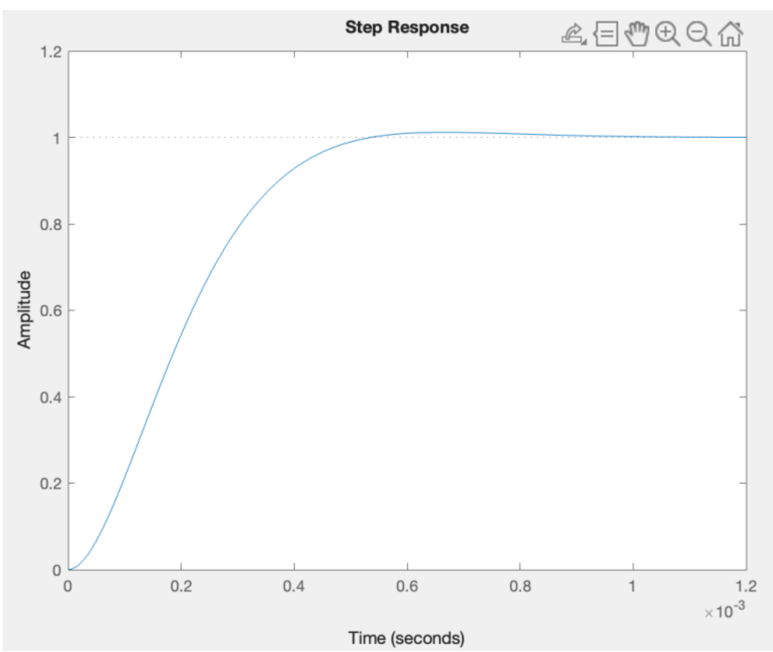


5.8.) For $R = 40\Omega, L = 3\ mH$ and $C = 5\ \mu F$, find the impulse response. Using step(H), find the step response of the system.

a. Plot and include Impulse Response



b. Plot and include step response



5.9.) Find the poles, zeros, and pole-zero map of the system. Is the system asymptotically stable, stable, or unstable? Why?

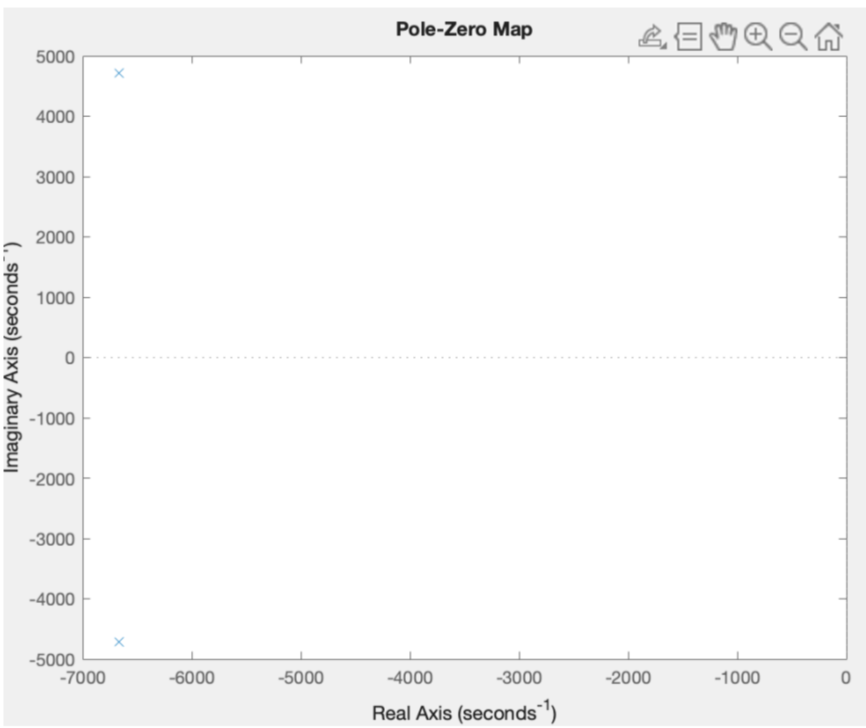
a. Include the poles and zeros found

p =

1.0e+03 *

-6.6667 + 4.7140i
-6.6667 - 4.7140i

b. Plot and include pole-zero map



c. Is the system asymptotically stable, stable, or unstable? Why?

- Asymptotically stable because impulse function approaches zero as time increases and there are only poles that are strictly less than zero

5.10.) Maintain the values of L and C as $L = 3\ mH$ and $C = 5\ \mu F$. Set the resistance value are to $R = 1\Omega$, $R = 2\Omega$, $R = 3\Omega$, and $R = 10\Omega$. What happens to the impulse response in terms of oscillations as the resistance varies? How does the pole magnitude vary as R is varied?

- The number of oscillations in the impulse response decreases as resistance increases. The pole magnitude remains constant

5.11.) Demonstrate that H(s) is unstable by finding the poles and plotting the impulse response. $H(s) = \frac{s^2-7s+12}{s^3+s^2-2}$

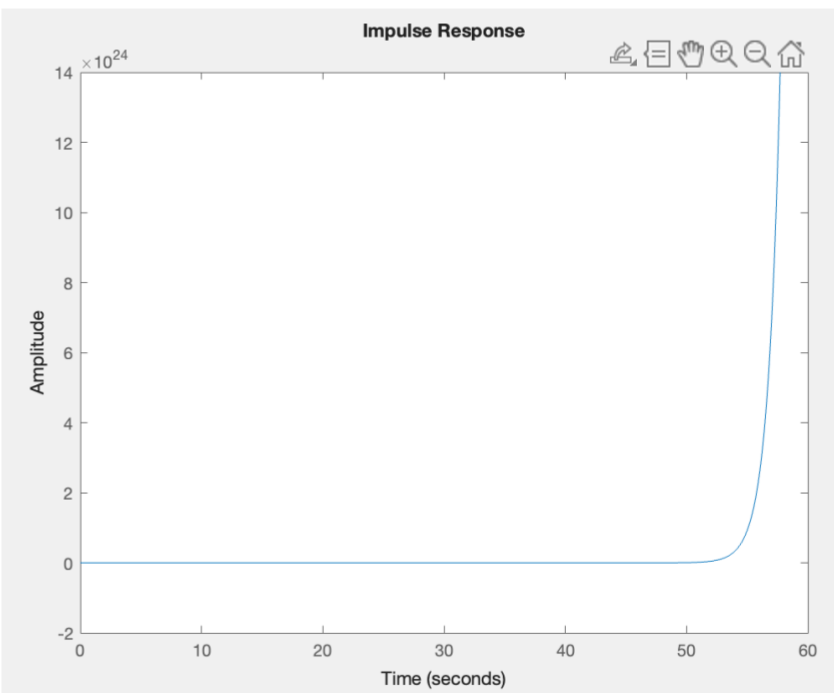
a. Include the poles and zeros found

p =

-1.0000 + 1.0000i
-1.0000 - 1.0000i
1.0000 + 0.0000i

z = 3,4

b. Plot and include impulse response



5.12.) Write $H(s)$ in the form described in Equation (3).

H(s) = (s – 3) (s – 4)

(s – 1) (s + 1 – j) (s + 1 + j)

5.13.) Perform pole-zero cancelation to get rid of the unstable pole and plot the impulse response of the new system).

a. Pole-Zero cancellation in the form of Equation 3.

H(s) = (s – 3) (s – 4)
-------------------- * (s-1) =
(s – 1) (s + 1 – j) (s + 1 + j)

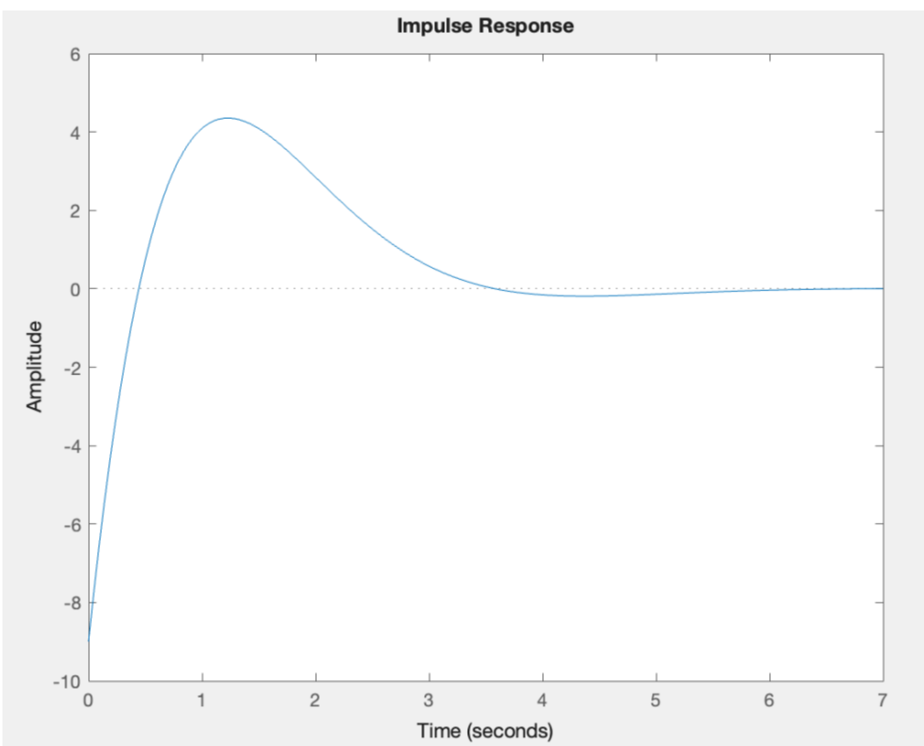
Hnew(s) = (s – 3) (s – 4)

(s + 1 – j) (s + 1 + j)

b. Pole-Zero cancellation in transfer function form (multiplied out as seen in #11).

H(s) = s^2 – 7s + 12
----
s^2 + 2s + 2

c. Plot and include the impulse response of the new system.



d. Now, is the system asymptotically stable, stable, or unstable? Is this what you expected to see?

    - System is asymptotically stable which would be expected because the pole greater than zero was removed and the real parts of all remaining poles are strictly less than zero.

Conclusions:

- I enjoyed learning about poles and zeros because they provide an easy-to-understand way of studying a system and its stability. No major issues as the code ran smoothly. No major improvements.